Physical Transport Phenomena 2
Reexamination
02-07-2013

Please use the provided paper sheets to write down the solutions of the problems. Write your name and student ID number on a first page and enumerate all subsequent pages. Do not forget to hand in your paperwork after the examination.

Problem 1

Velocity field is given by

$$\vec{v} = \begin{pmatrix} t/x^2 \\ y(t-1) \end{pmatrix}$$

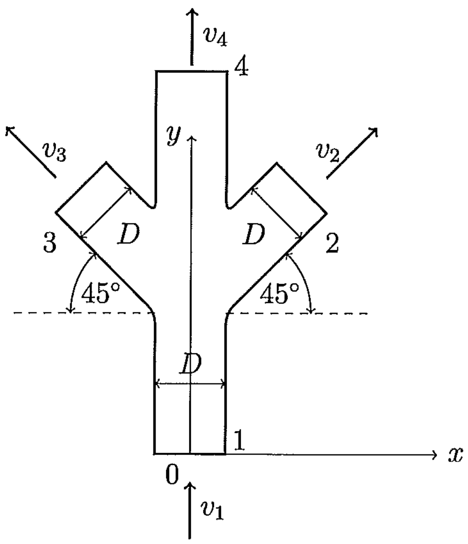
a. Is the flow compressible or incompressible?

b. Find the streamline that pass point (1, 1) at time t.

c. Find the trajectory that pass point (1, 1) at $t = 1$.

Problem 2

Water enters a pipe system shown in Figure through the section 1 and leaves through the sections 2, 3 and 4. The pressures at the sections 1 and 4 are 1 Bar and 2 Bar, respectively. The diameter of the pipes is 20 cm. The velocity at the section 1 is 20 m/s. The pressures in the sections 2 and 3 are the same. Water can be considered incompressible with the density $\rho = 1 \times 10^3\,\text{kg/m}^3$. Viscous dissipation and gravitational forces are negligible. The velocity profiles in all cross-sections can be considered uniform.



a. Find the velocity in the section 4.

b. Find the pressure in the section 2.

c. Find the forces acting by fluid on the pipe system in the x and y directions.

Problem 3

The mass flux J[kg/s] of metal molecules from a buffer gas on a spherical particle depends upon its radius R_0, the diffusion coefficient D and the difference $\Delta\rho_0$ between densities of the metal molecules on the particle and in the buffer gas.

a. How many nondimensional groups in this problem?

b. Using the dimensional analysis find a functional dependence of the mass flux upon other parameters.

c. Assuming that the densities of the metal molecules at the surface of the particle and in the buffer gas are ρ^* and ρ_0, respectively, find the density of metal molecules in the buffer gas as a function of the distance R from the particle center.

Problem 4

The fluid with density ρ, viscosity μ, heat conductivity λ and heat capacity C_p flows around a body. In the boundary layer, the fluid velocity and temperature profiles can be approximated by

$$u(y) = U_\infty \left(1 - \frac{y_u^3}{(y + y_u)^3}\right)$$
$$T(y) = T_0 + (T_\infty - T_0)\left(1 - \frac{y_t^3}{(y + y_t)^3}\right)$$

where y is the axis orthogonal to the surface of the body and the rest of the coefficients are constants.

a. Obtain an expression for the friction coefficient C_f as a function of the mentioned above constants and the fluid properties.

b. Obtain an expression for the heat transport coefficient h as a function of the mentioned above constants, and the fluid properties.

c. Using the integral measure, find the thickness of the velocity boundary layer.

1

2

$$\vec{x} = \vec{x}(t, \vec{x}_0) \tag{2.1}$$

$$\vec{v} = \vec{v}(\vec{x}, t) \tag{2.2}$$

$$\begin{aligned} \vec{v} &= \frac{\mathrm{d}\vec{x}(t, \vec{x}_0)}{\mathrm{d}t} \\ \vec{a} &= \frac{\mathrm{d}\vec{v}(t, \vec{x}_0)}{\mathrm{d}t} \end{aligned} \tag{2.3}$$

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} \tag{2.4}$$

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + (\vec{v} \cdot \vec{\nabla})c \tag{2.5}$$

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + v_i\frac{\partial c}{\partial x_i} \tag{2.6}$$

$$\frac{\mathrm{d}x}{u} = \frac{\mathrm{d}y}{v} = \frac{\mathrm{d}z}{w} \tag{2.7}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = u \; \frac{\mathrm{d}y}{\mathrm{d}t} = v \; \frac{\mathrm{d}z}{\mathrm{d}t} = w \tag{2.8}$$

$$Q = \int_S \vec{v} \cdot \vec{n} \,\mathrm{d}S \tag{2.9}$$

$$\dot{m} = \int_S \rho\vec{v} \cdot \vec{n} \,\mathrm{d}S \tag{2.10}$$

$$\bar{v} = \frac{Q}{S} \tag{2.11}$$

$$F = \int_S \rho\varphi\vec{v} \cdot \vec{n} \,\mathrm{d}S \tag{2.12}$$

3

$$\vec{f}_V = \frac{\mathrm{d}\vec{F}_V}{\mathrm{d}V} \tag{3.1}$$

$$\vec{f}_m = \frac{\mathrm{d}\vec{F}}{\mathrm{d}m} = \frac{\mathrm{d}\vec{F}_V}{\rho \mathrm{d}V} \tag{3.2}$$

$$\begin{aligned} \vec{F}_V &= \int_V \vec{f}_V \, \mathrm{d}V \\ \vec{F}_V &= \int_V \rho \vec{f}_m \, \mathrm{d}V \end{aligned} \tag{3.3}$$

$$\vec{f}_V = \rho \vec{g} \tag{3.4}$$

$$\vec{f}_V = \rho_e \vec{E} + \vec{j} \times \vec{B} \tag{3.5}$$

$$\vec{f}_m = -\left(\vec{a}_0 + \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right) + 2\vec{\Omega} \times \vec{v}\right) \tag{3.6}$$

$$\vec{F}_S = \int_S \vec{f}_S \, \mathrm{d}S \tag{3.7}$$

$$\begin{pmatrix} f_{S1} \\ f_{S_2} \\ f_{S3} \end{pmatrix} = \begin{bmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{bmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \tag{3.8}$$

$$\vec{f}_S = \hat{\tau} \vec{n} \tag{3.9}$$

$$\tau_{ik} = \tau_{ki} \tag{3.10}$$

$$\tau = \begin{bmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{bmatrix} \tag{3.11}$$

$$p_{gag} = p_{abs} - p_{atm} \tag{3.12}$$

4

$$\rho \vec{f}_m - \vec{\nabla} p = 0 \tag{4.1}$$

$$\begin{aligned} \rho \vec{f}_m - \rho \vec{a} - \vec{\nabla} p = 0 \\ \text{where } \vec{a} = \vec{a}_0 + \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \end{aligned} \tag{4.2}$$

$$p_2 - p_1 = -\rho g(z_2 - z_1) \tag{4.3}$$

$$p(h) = p_{atm} + \rho g h \tag{4.4}$$

$$p_2 = p_1 e^{-\frac{g\mu}{RT}(z_2 - z_1)} \tag{4.5}$$

$$\vec{F} = \int_S (-p\vec{n})\,\mathrm{d}S$$

$$\vec{M}_0 = \int_S \vec{r} \times (-p\vec{n})\,\mathrm{d}S \tag{4.6}$$

$$\left[\vec{r}_{cp} \times \vec{F}\right] = \vec{M}_0 \tag{4.7}$$

$$f_y = -\rho g V \tag{4.8}$$

5

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V(t)} \phi(\vec{X},t)\,\mathrm{d}V = \int_{V(t)} \frac{\partial \phi(\vec{X},t)}{\partial t}\,\mathrm{d}V + \int_{S(t)} \phi(\vec{X},t)\vec{w}\cdot\vec{n}\,\mathrm{d}S \tag{5.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_f(t)} \phi(\vec{X},t)\,\mathrm{d}V = \int_{V_f(t)} \frac{\partial \phi(\vec{X},t)}{\partial t}\,\mathrm{d}V + \int_{S_f(t)} \phi(\vec{X},t)\vec{v}\cdot\vec{n}\,\mathrm{d}S \tag{5.2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)} \phi(\vec{X},t)\,\mathrm{d}V = \int_{V_c(t)} \frac{\partial \phi(\vec{X},t)}{\partial t}\,\mathrm{d}V + \int_{S_c(t)} \phi(\vec{X},t)\overrightarrow{v^c}\cdot\vec{n}\,\mathrm{d}S \tag{5.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_f(t)} \phi(\vec{X},t)\,\mathrm{d}V = \frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)} \phi(\vec{X},t)\,\mathrm{d}V + \int_{S_c(t)} \phi(\vec{X},t)(\vec{v}\cdot - \overrightarrow{v^c})\vec{n}\,\mathrm{d}S \tag{5.4}$$

6

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_f(t)} \rho\,\mathrm{d}V = 0 \tag{6.1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)} \rho\,\mathrm{d}V + \int_{S_c(t)} \rho(\vec{v} - \overrightarrow{v^c})\cdot\vec{n}\,\mathrm{d}S = 0 \tag{6.2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}M(V_c) = \dot{M}_{in} - \dot{M}_{out} \tag{6.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_f(t)} \rho\vec{v}\,\mathrm{d}V = \int_{S_f(t)} \hat{\tau}\vec{n}\,\mathrm{d}S + \int_{V_F(t)} \rho\vec{f}_m\,\mathrm{d}V \tag{6.4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)}\rho\vec{v}\,\mathrm{d}V+\int_{S_c(t)}\rho\vec{v}\left[(\vec{v}-\vec{v^c})\cdot\vec{n}\right]\,\mathrm{d}S=\int_{S_c(t)}\hat{\tau}\vec{n}\,\mathrm{d}S+\int_{V_c(t)}\rho\vec{f}_m\,\mathrm{d}V \tag{6.5}$$

$$\frac{\mathrm{d}P(\vec{V}_c)}{\mathrm{d}t}=\dot{\vec{P}}_{in}-\dot{\vec{P}}_{out}+\sum\vec{F}_{ext} \tag{6.6}$$

$$\hat{\tau}=-p\hat{I}+\hat{\tau}' \tag{6.7}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_f(t)}\vec{r}\times\rho\vec{v}\,\mathrm{d}V=\int_{S_f(t)}\vec{r}\times\vec{f}_S\,\mathrm{d}S+\int_{V_f(t)}\vec{r}\times\rho\vec{f}_m\,\mathrm{d}V \tag{6.8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)}\vec{r}\times\rho\vec{v}\,\mathrm{d}V+\int_{S_c(t)}\vec{r}\times\rho\vec{v}\left[(\vec{v}-\vec{v^c})\cdot\vec{n}\right]\,\mathrm{d}S=\int_{S_c(t)}\vec{r}\times\vec{f}_S\,\mathrm{d}S+\int_{V_c(t)}\vec{r}\times\rho\vec{f}_m\,\mathrm{d}V \tag{6.9}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{H}(V_c)=\dot{\vec{H}}_{in}-\dot{\vec{H}}_{out}+\sum\vec{M}_{ext} \tag{6.10}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_f(t)}\rho\left(e+\frac{1}{2}v^2\right)\,\mathrm{d}V&=\int_{S_f(t)}(-p\vec{n}+\hat{\tau}'\vec{n})\cdot\vec{v}\,\mathrm{d}S\\&+\int_{V_f(t)}\rho\vec{f}\cdot\vec{v}\,\mathrm{d}V-\int_{S_f(t)}\vec{q}\cdot\vec{n}\,\mathrm{d}S+\int_{V_f(t)}\dot{q}_V\,\mathrm{d}V\end{aligned} \tag{6.11}$$

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)}\rho\left(e+\frac{1}{2}v^2\right)\,\mathrm{d}V+\int_{S_c(t)}\rho\left(e+\frac{1}{2}v^2\right)\left[(\vec{v}-\vec{v^c})\cdot\vec{n}\right]\,\mathrm{d}S\\&=\int_{S_c(t)}(-p\vec{n}+\hat{\tau}'\vec{n})\cdot\vec{v}\,\mathrm{d}S+\int_{V_c(t)}\rho\vec{f}\cdot\vec{v}\,\mathrm{d}V-\int_{S_c(t)}\vec{q}\cdot\vec{n}\,\mathrm{d}S+\int_{V_c(t)}\dot{q}_V\,\mathrm{d}V\end{aligned} \tag{6.12}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}E(V_c)=\dot{E}_{in}-\dot{E}_{out}+\dot{W}_{ext}+\dot{Q}_{in} \tag{6.13}$$

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)}\rho\left(e+\frac{1}{2}v^2+U\right)\,\mathrm{d}V+\int_{S_c(t)}\rho\left(e+\frac{1}{2}v^2+U\right)\left[(\vec{v}-\vec{v^c})\cdot\vec{n}\right]\,\mathrm{d}S\\&=\int_{S_c(t)}(-p\vec{n}+\hat{\tau}'\vec{n})\cdot\vec{v}\,\mathrm{d}S-\int_{S_c(t)}\vec{q}\cdot\vec{n}\,\mathrm{d}S+\int_{V_c(t)}\dot{q}_V\,\mathrm{d}V\end{aligned} \tag{6.14}$$

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)}\rho\frac{1}{2}v^2\,\mathrm{d}V+\int_{S_c(t)}\rho\frac{1}{2}v^2\left[(\vec{v}-\vec{v^c})\cdot\vec{n}\right]\,\mathrm{d}S\\&=\int_{S_c(t)}(-p\vec{n}+\hat{\tau}'\vec{n})\cdot\vec{v}\,\mathrm{d}S+\int_{V_c(t)}\rho\vec{f}_m\cdot\vec{v}\,\mathrm{d}V+\int_{V_c(t)}p\vec{\nabla}\cdot\vec{v}\,\mathrm{d}V-\int_{V_c(t)}\phi_v\,\mathrm{d}V\end{aligned} \tag{6.15}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}E^{mec}(V_c) = \dot{E}_{in}^{mec} - \dot{E}_{out}^{mec} + \dot{W}_{ext} + \dot{W}_{exp} - D_v \tag{6.16}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)}\rho\left(\frac{1}{2}v^2+U\right)\mathrm{d}V + \int_{S_c(t)}\rho\left(\frac{1}{2}v^2+U\right)\left[(\vec{v}-\vec{v^c})\cdot\vec{n}\right]\mathrm{d}S \\ = \int_{S_c(t)}(-p\vec{n}+\hat{\tau}'\vec{n})\cdot\vec{v}\,\mathrm{d}S + \int_{V_c(t)}p\vec{\nabla}\cdot\vec{v}\,\mathrm{d}V - \int_{V_c(t)}\phi_v\,\mathrm{d}V\end{aligned} \tag{6.17}$$

$$\frac{1}{2}v^2 + \frac{p}{\rho} + U = C_{sl} \tag{6.18}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)}\rho e\,\mathrm{d}V + \int_{S_c(t)}\rho e\left[(\vec{v}-\vec{v^c})\cdot\vec{n}\right]\mathrm{d}S \\ = -\int_{S_c(t)}p\vec{\nabla}\cdot\vec{v}\,\mathrm{d}S + \int_{V_c(t)}\phi_v\,\mathrm{d}V - \int_{S_c(t)}\vec{q}\cdot\vec{n}\,\mathrm{d}S + \int_{V_c(t)}\dot{q}_V\,\mathrm{d}V\end{aligned} \tag{6.19}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}E_{int}(V_c) = \dot{E}_{in}^{int} - \dot{E}_{out}^{int} - \dot{W}_{exp} + D_v + \dot{Q}_{in} \tag{6.20}$$

$$\begin{aligned}\sum\nolimits_{A=1}^{n_{esp}} Y_A = 1 \\ \rho = \sum\nolimits_{A=1}^{n_{esp}} \rho_A \\ \rho_A = \rho Y_A\end{aligned} \tag{6.21}$$

$$\begin{aligned}\sum\nolimits_{A=1}^{n_{esp}} X_A = 1 \\ c = \sum\nolimits_{A=1}^{n_{esp}} c_A \\ c_A = c X_A\end{aligned} \tag{6.22}$$

$$\rho_A = M_A c_A \tag{6.23}$$

$$\rho = \sum\nolimits_{A=1}^{n_{esp}} \rho_A = \sum\nolimits_{A=1}^{n_{esp}} M_A c_A = c\sum\nolimits_{A=1}^{n_{esp}} M_A X_A = cM \tag{6.24}$$

$$Y_A = \frac{\rho_A}{\rho} = \frac{c_A M_A}{\sum_{A=1}^{n_{esp}} c_B M_B} = \frac{X_A M_A}{\sum_{A=1}^{n_{esp}} X_B M_B} = \frac{X_A M_A}{M} \tag{6.25}$$

$$X_A = \frac{M Y_A}{M_A} = \frac{Y_A/M_A}{\sum_{A=1}^{n_{esp}} Y_B/M_B} \tag{6.26}$$

$$\vec{v} = \frac{\sum_{A=1}^{n_{esp}} \rho_A \vec{v}_A}{\rho} = \sum\nolimits_{A=1}^{n_{esp}} Y_A \vec{v}_A \tag{6.27}$$

$$\vec{v^m} = \frac{\sum_{A=1}^{n_{esp}} c_A \vec{v}_A}{c} = \sum\nolimits_{A=1}^{n_{esp}} X_A \vec{v}_A \tag{6.28}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_{cA}(t)}\rho_A\,\mathrm{d}V = \int_{V_{cA}(t)}\dot{\omega}_A\,\mathrm{d}V \tag{6.29}$$

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)}\rho_A\,\mathrm{d}V + \int_{S_c(t)}\rho_A\left[(\vec{v}-\overrightarrow{v^c})\cdot\vec{n}\right]\,\mathrm{d}S\\ &= -\int_{S_c(t)}\rho_A\left[(\vec{v}_A-\vec{v})\cdot\vec{n}\right]\,\mathrm{d}S + \int_{V_{cA}(t)}\dot{\omega}_A\,\mathrm{d}V\end{aligned} \tag{6.30}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}M_A(V_c) = \dot{M}_{A,in} - \dot{M}_{A,out} + J_{A,in} - J_{A,out} + \dot{G}_A \tag{6.31}$$

$$\vec{j}_A = \rho_A(\vec{v}_A - \vec{v}) \tag{6.32}$$

$$\vec{V}_A = (\vec{v}_A - \vec{v}) = \frac{\vec{j}_A}{\rho_A} \tag{6.33}$$

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)}c_A\,\mathrm{d}V + \int_{S_c(t)}c_A\left[(\vec{v}-\overrightarrow{v^c})\cdot\vec{n}\right]\,\mathrm{d}S\\ &= -\int_{S_c(t)}c_A\left[(\vec{v}_A-\vec{v})\cdot\vec{n}\right]\,\mathrm{d}S + \int_{V_{cA}(t)}\dot{\omega}'_A\,\mathrm{d}V\end{aligned} \tag{6.34}$$

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)}c_A\,\mathrm{d}V + \int_{S_c(t)}c_A\left[(\overrightarrow{v^m}-\overrightarrow{v^c})\cdot\vec{n}\right]\,\mathrm{d}S\\ &= -\int_{S_c(t)}c_A\left[(\vec{v}_A-\overrightarrow{v^m})\cdot\vec{n}\right]\,\mathrm{d}S + \int_{V_{cA}(t)}\dot{\omega}'_A\,\mathrm{d}V\end{aligned} \tag{6.35}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V_c(t)}\mathrm{d}V + \int_{S_c(t)}\left[(\vec{v}-\overrightarrow{v^c})\cdot\vec{v}\right]\mathrm{d}S = 0 \tag{6.36}$$

7

$$\tau'_{xy} = \mu\frac{\mathrm{d}u}{\mathrm{d}y} \tag{7.1}$$

$$\tau'_{ij} = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \lambda\frac{\partial v_k}{\partial x_k}\delta_{ij} \tag{7.2}$$

$$\nu = \frac{\mu}{\rho} \tag{7.3}$$

$$\tau' = \mu_0\left|\frac{\mathrm{d}u}{\mathrm{d}y}\right|^{n-1}\frac{\mathrm{d}u}{\mathrm{d}y} \tag{7.4}$$

$$\begin{cases} \tau' > \tau'_0, & \tau' = \tau'_0 + \mu_0 \left|\frac{\mathrm{d}u}{\mathrm{d}y}\right|^{n-1} \frac{\mathrm{d}u}{\mathrm{d}y} \\ \tau' < \tau'_0, & \frac{\mathrm{d}u}{\mathrm{d}y} = 0 \end{cases} \tag{7.5}$$

$$q_y = -k\frac{\partial T}{\partial y} \tag{7.6}$$

$$\vec{q} = -\hat{k}\vec{\nabla}T \tag{7.7}$$

$$\vec{j}_A = -\rho D_{AB}\vec{\nabla}Y_A \tag{7.8}$$

$$\vec{j'}_A = -\frac{\rho D_{AB}}{M_A}\vec{\nabla}Y_A \tag{7.9}$$

$$\vec{j}_A^m = -\frac{cD_{AB}}{M_A}\vec{\nabla}X_A \tag{7.10}$$

$$\vec{j'}_A^m = -cD_{AB}\vec{\nabla}X_A \tag{7.11}$$

$$D = \frac{1}{6}Nm_p v_a \lambda \tag{7.12}$$

$$\mu = \frac{k}{c_p} = \rho D_{AA} \tag{7.13}$$

8

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0 \tag{8.1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_i)}{\partial x_i} = 0 \tag{8.2}$$

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\vec{\nabla}\cdot\vec{v} \tag{8.3}$$

$$\vec{\nabla}\cdot\vec{v} = 0 \tag{8.4}$$

$$\frac{\partial(\rho v_i)}{\partial t} + \frac{\partial(\rho v_i v_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau'_{ij}}{\partial x_j} + \rho f_{m,i} \tag{8.5}$$

$$\frac{\partial(\rho\vec{v})}{\partial t} + \vec{\nabla}\cdot(\rho\widehat{\vec{v}\vec{v}}) = -\vec{\nabla}p + \vec{\nabla}\cdot\hat{\tau}' + \rho\vec{f_m} \tag{8.6}$$

$$\frac{\partial(\rho v_i)}{\partial t} + \frac{\partial(\rho v_i v_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 v_i}{\partial {x_j}^2} + \rho f_{m,i} \tag{8.7}$$

$$\frac{\partial}{\partial t}\left(e+\frac{1}{2}v^2+U\right)+\vec{\nabla}\cdot\left(\rho\left(e+\frac{1}{2}v^2+U\right)\vec{v}\right)=\vec{\nabla}\cdot(\hat{t}\vec{v})-\vec{\nabla}\cdot\vec{q}+\dot{q}_v \quad (8.8)$$

$$\frac{\partial}{\partial t}\left(e+\frac{1}{2}v^2+U\right)+\frac{\partial}{\partial x_j}\left(\rho\left(e+\frac{1}{2}v^2+U\right)v_j\right)=\frac{\partial}{\partial x_j}(\tau_{ij}v_i)-\frac{\partial}{\partial x_j}q_j+\dot{q}_v \quad (8.9)$$

$$\phi_v=\tau'_{ij}\frac{\partial v_i}{\partial x_j} \quad (8.10)$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v_i^2\right)+\frac{\partial}{\partial x_j}\left(\frac{1}{2}\rho v_j v_i^2\right)=\frac{\partial}{\partial x_j}(\tau_{ij}v_i)+p\frac{\partial v_i}{\partial x_i}-\phi_v+v_i\rho f_{m,i} \quad (8.11)$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2\right)+\vec{\nabla}\cdot\left(\frac{1}{2}\rho v^2\vec{v}\right)=\vec{\nabla}\cdot(\hat{\tau}\vec{v})+p\vec{\nabla}\cdot\vec{v}-\phi_v+\rho\vec{f}_m\cdot\vec{v} \quad (8.12)$$

$$\frac{\partial}{\partial t}(\rho e)+\vec{\nabla}\cdot(\rho e\vec{v})=-p\vec{\nabla}\cdot\vec{v}+\phi_v-\vec{\nabla}\cdot\vec{q}+\dot{q}_v \quad (8.13)$$

$$\frac{\partial}{\partial t}(\rho e)+\frac{\partial}{\partial x_j}(\rho e v_j)=-p\frac{\partial v_j}{\partial x_j}+\phi_v-\frac{\partial q_j}{\partial x_j}+\dot{q}_v \quad (8.14)$$

$$\rho c_v\frac{\partial T}{\partial t}+\rho c_v v_j\frac{\partial T}{\partial x_j}=-p\frac{\partial v_j}{\partial x_j}+\phi_v+k\frac{\partial^2 T}{\partial {x_j}^2}+\dot{q}_v \quad (8.15)$$

$$\rho c_v\frac{\partial T}{\partial t}+\rho c_v v_j\frac{\partial T}{\partial x_j}=\phi_v+k\frac{\partial^2 T}{\partial {x_j}^2}+\dot{q}_v \quad (8.16)$$

$$\rho c_p\frac{\partial T}{\partial t}+\rho c_p v_j\frac{\partial T}{\partial x_j}=\phi_v+k\frac{\partial^2 T}{\partial {x_j}^2}+\dot{q}_v \quad (8.17)$$

$$\frac{\partial}{\partial t}(\rho h)+\vec{\nabla}\cdot(\rho h\vec{v})=\frac{\partial p}{\partial t}+\vec{v}\cdot\vec{\nabla}p+\phi_v-\vec{\nabla}\cdot\vec{q}+\dot{q}_v \quad (8.18)$$

$$\frac{\partial}{\partial t}(\rho h)+\frac{\partial}{\partial x_j}(\rho h v_j)=\frac{\partial p}{\partial t}+v_j\frac{\partial p}{\partial x_j}+\phi_v-\frac{\partial q_j}{\partial x_j}+\dot{q}_v \quad (8.19)$$

$$\frac{\partial}{\partial t}(\rho s)+\frac{\partial}{\partial x_j}(\rho s v_j)-\frac{\partial}{\partial x_j}\left(\frac{k}{T}\frac{\partial T}{\partial x_j}\right)-\frac{\dot{q}_v}{T}=\frac{\phi_v}{T}+\frac{k}{T^2}\left(\frac{\partial T}{\partial x_j}\right)^2 \quad (8.20)$$

$$\frac{\partial}{\partial t}(\rho Y_A)+\vec{\nabla}\cdot(\rho Y_A\vec{v})=-\vec{\nabla}\cdot\vec{j}_A+\dot{\omega}_A \quad (8.21)$$

$$\frac{\partial}{\partial t}(\rho Y_A)+\frac{\partial}{\partial x_i}(\rho Y_A v_i)=-\frac{\partial}{\partial x_i}(j_{Ai})+\dot{\omega}_A \quad (8.22)$$

$$\frac{\partial}{\partial t}(\rho_A)+\frac{\partial}{\partial x_i}(\rho_A v_i)=-\frac{\partial}{\partial x_i}(j_{Ai})+\dot{\omega}_A \quad (8.23)$$

$$\frac{\partial Y_A}{\partial t} + v_i \frac{\partial Y_A}{\partial x_i} = D \frac{\partial^2 Y_A}{\partial {x_i}^2} + \frac{\dot{\omega}_A}{\rho} \tag{8.24}$$

$$\frac{\partial c_A}{\partial t} + v_i \frac{\partial c_A}{\partial x_i} = D \frac{\partial^2 c_A}{\partial {x_i}^2} \frac{\dot{\omega}_A}{M_A} \tag{8.25}$$

9

$$Re = \frac{\rho U D}{\mu} \quad \text{Reynolds number} \tag{9.2}$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} \tag{9.3}$$

$$F_D = 3\pi\mu U D \tag{9.4}$$

$$C_D = \frac{24}{Re} \tag{9.5}$$

10

$$\phi = \phi_0 \phi' \tag{10.1}$$

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= \frac{\phi_0}{t_0} \frac{\partial \phi'}{\partial t'} \\ \frac{\partial \phi}{\partial x} &= \frac{\phi_0}{l_0} \frac{\partial \phi'}{\partial x'} \end{aligned} \tag{10.2}$$

$$\frac{\partial \phi}{\partial t} \approx \frac{\phi_0}{t_0} \text{ and } \frac{\partial \phi'}{\partial t'} \approx 1 \tag{10.3}$$

$$S = \frac{l_0}{v_0 t_0} \quad \text{Strouhal number} \tag{10.4}$$

$$S \frac{\partial \rho'}{\partial t'} + \vec{\nabla}' \cdot (\rho' \vec{v}') = 0 \tag{10.5}$$

$$\begin{aligned} Eu &= \frac{\Delta p_0}{\rho_0 v_0^2} \quad \text{Euler number} \\ Re &= \frac{\rho_0 v_0 l_0}{\mu_0} \quad \text{Reynolds number} \\ Fr &= \frac{v_0^2}{g l_0} \quad \text{Froude number} \end{aligned} \tag{10.6}$$

$$S\frac{\partial\rho\vec{v}'}{\partial t} + \vec{\nabla}'\cdot(\rho'\widehat{\vec{v}'\vec{v}'}) = -Eu\vec{\nabla}'p + \frac{1}{Re}\vec{\nabla}'\cdot(\hat{\tau}')' + \frac{1}{Fr}\rho'\vec{f}'_m \tag{10.7}$$

$$C_D = \frac{\tau'_0}{\frac{1}{2}\rho v_0^2} \quad \text{Drag coefficient} \tag{10.8}$$

$$\begin{aligned} Pe &= \frac{\rho_0 c_{p0} v_0 l_0}{k_0} \quad \text{Peclet number} \\ Ec = Re\frac{\mu_0 v_0}{\rho_0 c_{p0}\Delta T_0 l_0} &= \frac{v_0^2}{c_{p0}\Delta T_0} \quad \text{Eckert number} \end{aligned} \tag{10.9}$$

$$S\rho' c'_p \frac{\partial T'}{\partial t'} + \rho' c'_p \vec{v}' \cdot \vec{\nabla}' T' = \frac{1}{Pe}k'\nabla'^2 T' + \frac{Ec}{Re}\phi'_v \tag{10.10}$$

$$Nu = \frac{q_0 l_0}{k_0 \Delta T_0} \tag{10.11}$$

$$\begin{aligned} Pe_{II} &= \frac{v_0 l_0}{D_{A0}} \quad \text{Peclet II number} \\ Da_I &= \frac{\dot{\omega}_{A0} l_0}{\rho_{A0} v_0} \quad \text{Damkohler I number} \end{aligned} \tag{10.12}$$

$$S\frac{\partial\rho'_A}{\partial t'} + \vec{\nabla}'\cdot(\rho'_A\vec{v}') = -\frac{1}{Pe_{II}}\vec{\nabla}\cdot\vec{j}_A + Da_I\dot{\omega}'_A \tag{10.13}$$

$$Nu_m = Sh = \frac{j_{A0} l_0}{D_{A0}\rho_{A0}} \quad \text{Mass Nusselt or Sherwood number} \tag{10.14}$$

$$Pr = \frac{Pe}{Re} = \frac{\mu_0 c_{p0}}{k_0} \quad \text{Prandtl number} \tag{10.15}$$

$$Sc = \frac{Pe_{II}}{Re} = \frac{\mu_0}{\rho_0 D_{A0}} \quad \text{Schmidt number} \tag{10.16}$$

$$Le = \frac{Pe_{II}}{Pe} = \frac{k_0}{\rho_0 c_{p0} D_{A0}} \quad \text{Lewis number} \tag{10.17}$$

$$Ma = \frac{v_0}{c_0} \quad \text{Mach number} \tag{10.18}$$

11

$$
\begin{aligned}
\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}&=0\\
u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}&=-\frac{1}{p}\frac{\mathrm{d}p_e}{\mathrm{d}x}+\nu\frac{\partial^2 u}{\partial y^2}\\
u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}&=\alpha\frac{\partial^2 T}{\partial y^2}\\
u\frac{\partial c_A}{\partial x}+v\frac{\partial c_A}{\partial y}&=D_A\frac{\partial^2 c_A}{\partial y^2}
\end{aligned}
\tag{11.1}
$$

$$\delta\approx\sqrt{\frac{\nu L}{U_e}}=\frac{L}{\sqrt{\frac{U_e L}{\nu}}}=\frac{L}{\sqrt{Re}} \tag{11.2}$$

$$\delta_T\approx\frac{L}{\sqrt{Pe}},\quad Pr\ll 1 \tag{11.3}$$

$$\delta_t\approx\frac{L}{Re^{\frac{1}{2}}Pr^{\frac{1}{3}}}=L\frac{Pr^{\frac{1}{6}}}{\sqrt{Pe}},\quad Pr\gg 1 \tag{11.4}$$

$$\delta_c\approx\frac{L}{\sqrt{Pe_{II}}},\quad Sc\ll 1 \tag{11.5}$$

$$\delta_c\approx\frac{L}{Re^{\frac{1}{2}}Sc^{\frac{1}{3}}}=L\frac{Sc^{\frac{1}{6}}}{\sqrt{Pe_{II}}},\quad Sc\gg 1 \tag{11.6}$$

$$\delta=\int_0^\infty\left(1-\frac{u(y)}{U_e}\right)\,\mathrm{d}y \tag{11.7}$$

12

$$\bar{K}=\frac{1}{A}\int_A K\,\mathrm{d}A \tag{12.1}$$

$$\frac{\mathrm{d}F}{\mathrm{d}A}=-K_\tau\rho V_\infty \tag{12.2}$$

$$K_\tau=\frac{\tau_0'}{\rho V_\infty} \tag{12.3}$$

$$C_f=\Phi(Re) \tag{12.4}$$

$$C_f = \frac{\tau_0'}{\frac{1}{2}\rho V_\infty^2} \tag{12.5}$$

$$K_\tau = \frac{1}{2} V_\infty C_f \tag{12.6}$$

$$u = \frac{p_1 - p_2}{4\mu L}(R^2 - r^2) \tag{12.7}$$

$$C_f = \frac{\tau_0'}{\frac{1}{2}\rho V_\infty^2} = \frac{4V_\infty \mu}{R} / \frac{1}{2}\rho V_\infty^2 = \frac{8\mu}{\rho V_\infty R}\frac{16\mu}{\rho V_\infty D} = \frac{16}{Re_D} \tag{12.8}$$

$$h_f = \frac{\Delta p*}{\rho g} = \lambda\left(\frac{\epsilon}{D}, Re_D\right)\frac{L}{D}\frac{U^2}{2g} \tag{12.9}$$

$$C_f = \frac{\lambda}{4} \tag{12.10}$$

$$h_f = \frac{\Delta p^*}{\rho g} = K_S \frac{U^2}{2g} \tag{12.11}$$

$$C_f = \frac{D}{L}\frac{K_S}{4} \tag{12.12}$$

$$\frac{\mathrm{d}\dot{Q}}{\mathrm{d}A} = h(T_0 - T_\infty) \tag{12.13}$$

$$\dot{Q} = \bar{h}(T_0 - T_\infty)A \tag{12.14}$$

$$St = \frac{h}{\rho V c_p} \quad \text{Stanton number} \tag{12.15}$$

$$St = \frac{h}{\rho V c_p} = \phi(Re, Pe) \tag{12.16}$$

$$Nu = \frac{hl}{k} \quad \text{Nusselt number} \tag{12.17}$$

$$Nu = \tilde{\phi}(Re, Pr) \quad \text{Forced convection} \tag{12.18}$$

$$Gr = \frac{\rho^2 g \beta \Delta T L^3}{\mu^2} \quad \text{Grashof number} \tag{12.19}$$

$$Ra = Gr Pr = \frac{c_p \rho^2 g \beta \Delta T L^3}{\mu k} = \frac{\rho g \beta \Delta T L^3}{\mu \alpha} \quad \text{Rayleigh number} \tag{12.20}$$

$$Nu = \tilde{\phi}'(Pr, Gr) = \tilde{\phi}''(Pr, Ra) \quad \text{Natural convection} \tag{12.21}$$

$$\dot{Q} = \bar{h}(T_0 - T_\infty)A = \bar{Nu}\frac{k}{l}(T_0 - T_\infty)A \tag{12.22}$$

$$j_A = \frac{\mathrm{d}J_A}{\mathrm{d}A} = h_m(\rho_{A0} - \rho_{A\infty}) \tag{12.23}$$

$$J_A = \bar{h}_m(\rho_{A0} - \rho_{A\infty})A \tag{12.24}$$

$$\frac{\mathrm{d}J_A}{\mathrm{d}A} = h_Y(Y_{A0} - Y_{A\infty}) \tag{12.25}$$

$$\frac{\mathrm{d}J_A^m}{\mathrm{d}A} = h_c(c_{A0} - c_{A\infty}) = h_X(X_{A0} - X_{A\infty}) \tag{12.26}$$

$$Sh = Nu_m = \frac{h_m l}{D_{AB}} \quad \text{Sherwood number} \tag{12.27}$$

$$Nu_m = Sh = \tilde{\phi}'(Re, Pe_{II} = \tilde{\phi}''(Re, Sc) \quad \text{Forced convection} \tag{12.28}$$

$$Gr_m = \frac{\rho_0 g \Delta\rho_A l^3}{\mu^2} \quad \text{Concentration Grashof number} \tag{12.29}$$

$$Nu_m = Sh = \tilde{\phi}'(Gr_m, Sc) \quad \text{Natural convection} \tag{12.30}$$

$$h_m^g(f(\rho_{A0}^l - f(\rho A\infty^g) = h_m^l(\rho_{A\infty}^l - \rho_{A0}^l) \tag{12.31}$$

$$\frac{Re}{2}C_f = Nu = Sh \tag{12.32}$$

$$\begin{aligned} \frac{Re}{2}C_f &= NuPr^{-\frac{1}{3}} \quad 0.6 < Pr < 60 \\ \frac{Re}{2}C_f &= ShSc^{-\frac{1}{3}} \quad 0.6 < Sc < 6000 \end{aligned} \tag{12.33}$$

$$Sh = Nu\left(\frac{Sc}{Pr}\right)^{\frac{1}{3}} = NuLe^{\frac{1}{3}} \begin{cases} 0.6 < Pr < 60 \\ 0.5 < Sc < 6000 \end{cases} \tag{12.34}$$